USING INTERPRETIVE FRAMES TO INFORM SELECTIONS OF ARTIFACTS OF STUDENT THINKING

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An important decision that professional development (PD) facilitators must make when preparing for activities with teachers is to select an appropriate tool for the intended learning goals of the PD (Sztajn, Borko, & Smith, 2017). One important and prevalent tool is artifacts of student thinking (e.g. Jacobs & Philipp, 2004). In this paper we add to the literature on artifact selection for professional development by discussing the affordances and constraints of different written artifacts of student thinking. Through a professional noticing assessment, we examine the interpretive frames (Sherin & Russ, 2014) that were invoked by 72 secondary teachers regarding 6 students' written strategies to proportional reasoning tasks. We characterize different ways teachers might make sense of different artifacts of student thinking, and discuss for what purposes PD facilitators might select particular written solutions.

Keywords: Written Artifacts of Student Thinking, Secondary Teachers, Interpretive Frames, Professional Development

In their review of research about professional development (PD) of mathematics teachers, Sztajn, Borko, and Smith (2017) found two striking similarities across the programs illustrated in the research. First, there was a similar vision for effective mathematics teaching across the programs, namely that teaching should include facilitation of interactions among students as they engage in rich mathematical tasks, that teachers should elicit and use students' emerging ideas to reach a mathematical goal, and that an overarching goal should be advancing understanding for all students. Second, the PDs were structured in similar ways; leaders *actively engaged* teachers in activities (as opposed to lecturing) about mathematics content and the teaching and learning of said content, and programs spanned many months and included many hours of work. Additionally, many PD programs focus on students' mathematical ideas as a way to support teacher learning, through the use of video artifacts (e.g. Sherin & van Es, 2005) or written artifacts (e.g. Jacobs & Philipp, 2004; Kazemi & Franke, 2004). Herein, we focus on written student work, given its prominent use in PD and its accessibility for teachers. To support PD leaders, we investigate the affordances and constraints of different features of written student work and how they might influence what teachers notice.

There are many ways teachers might interact with written student work. For example, National School Reform Faculty (2014) has over 200 different protocols that teachers and PD leaders can use to engage with students' written work, each supporting different discussions about a variety of pedagogical topics. However, little is known about *which* strategies teachers might be interested in analyzing and discussing (for selection criteria for videos of student thinking, see Sherin, Linsenmeier, & van Es, 2009). Hence, we investigated teachers' perceptions of six different written solution strategies, with the aim of informing future PD leaders as they select particular strategies to engage their teachers.

Conceptual Framework

In order to understand teachers' perceptions of the different artifacts, we draw upon the construct of *interpretive frames* (Sherin & Russ, 2014). Sherin and Russ defined interpretive

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frames as "structures that describe the ways in which a teacher's selective attention both grows out of and informs a teacher's knowledge-based reasoning, and vice-versa" (p. 3). In particular, what teachers notice is both contextual and interdependent. That is, what teachers notice depends on what they noticed previously, and is influenced by their beliefs, values, and/or knowledge. Teachers actively (yet tacitly) create particular frames through which they view their surroundings in order to make sense of their surroundings.

In their chapter, Sherin and Russ (2014) identified 13 different types of interpretive frames that teachers' created while making sense of videos of classroom lessons. Consider the following examples. In a video of a student loudly tapping their pen on a desk, a teacher viewing the event through an *evaluative* frame might claim the student is off task, and must be a bad student. Alternatively, a teacher viewing the event through an *affective* frame might express an emotional reaction to the event, and talk about how pen tapping irritates him/her. As another example, a teacher viewing the event through a *principle* frame might see the pen tapping as evidence the student is off-task, and describe this instance by citing a principle such as "students tend to act disruptively when they don't have access to the task." Importantly, each frame supported a different *way* of noticing the event, and teachers' perceptions of the event provide evidence of the creation of particular frames. In this study we focus on investigating two types of frames teachers create: *narrative* frames and *personal* frames. Narrative frames refer to when teachers simply describe what they notice, which may or may not include identifying causal relations. Personal frames refer to when teachers experience a personal connection to what they notice, which may include emotional reactions to what they notice, or desires to interact with the student/strategy in some way. We focus on these two frames because we believe they have much potential to influence the depth at which teachers engage with students' ideas.

Methods

To understand how different pieces of written student work influence what teachers notice, 72 practicing and prospective secondary teachers completed a survey. In the survey teachers responded to prompts about six different strategies for solving two proportional reasoning tasks. We then analyzed teachers' responses by identifying *when* teachers created particular interpretive frames, and *for which strategies* teachers created these frames (Sherin & Russ, 2014). In the next four sections we describe the participants, survey, strategies, and our analysis.

Participants

The secondary teachers in this study come from two teacher populations in the southwestern United States. First, 30 prospective secondary mathematics teachers were recruited from a large urban university. All intended to become secondary teachers, and none had begun the post-baccalaureate credential program offered at that university, nor student teaching. Second, 42 experienced practicing teachers (grades 6 - 12) were recruited from the southwestern region of the United States. All teachers had at least 4 years of experience teaching, and an average of 13.1 years of teaching experience.

We recognize that there are important differences among the two teacher groups that may influence what the teachers notice; however, parsing out these differences is beyond the scope of this paper. Our purpose in this paper is to investigate and discuss some of the ways different artifacts of student thinking pique secondary teachers' curiosity. For that reason, in the rest of the paper the term "teacher" will refer to both prospective and practicing teachers.

Survey

The survey consisted of three parts. In part 1, teachers first solved a missing-value proportional reasoning task, and then considered three student's written strategies for solving that task (see table 1). Next, teachers responded to three professional noticing prompts about the

Table 1: Tasks, Students, Strategies, and Descriptions

Student	Strategy	Our Description of the Strategy
	Task: Each day, 6 mice eat 18 food pellets. How many food pellets do 24 mice eat?	
A	$6\overline{)18}$ 3 24 x 3 62 food pellets	Student divides 18 pellets by 6 to find a unit rate of 3 pellets per mouse. Student multiplies unit rate by 24 mice. Student makes a mistake when multiplying 24x3.
B	$\frac{18}{6} = \frac{x}{24}$ $\frac{432}{6} = \frac{6x}{6}$ $\boxed{72 = x}$	Student uses the traditional cross-multiplication strategy, setting up pellets on top and mice on bottom of the fraction. Student manipulates equation correctly.
C	24 ÷ 6 = 4 18 x 4 = 72	Student divides 24 mice by 6 mice to find the number of groups of 6 mice in 24. There are 4 groups of 6 mice, so there should be 4 groups of 18 pellets.
	Task: Each day, 8 caterpillars eat 12 leaves. How many leaves do 20 caterpillars eat?	
D	$8 \div 12 = 1\frac{1}{2}$ $8 \div 8 = 1$ $12 - 8 = 4 = \frac{1}{2}$ of 8 $1\frac{1}{2} \times 20 = X$ 30 leaves $1 \times 20 = 20$ $20 \times 1 = 20$ $\frac{20}{2} = 10$ $20 + 10 = 30$	Student divides 12 leaves by 8 caterpillars (mistakenly writes 8 divided by 12), to find each caterpillar eats 1.5 leaves. The student divides in a non-standard way. The student then multiplies 1.5 leaves x 20 caterpillars, using the distributive property to help with the multiplication.
E	8 caterpillers x 3 = 24 caterpillars 12 x 3 = 36 leaves 36 − 4 = 32 24 − 4 = 20 Caterpillars | leaves 20 − 32	Student multiplies both 8 caterpillars and 12 leaves by 3, apparently scaling up the ratio to 24 caterpillars and 36 leaves. The student subtracts 4 from both quantities to obtain the 20 caterpillars, and arrives at 32 leaves. This last part exhibits additive reasoning, and is not correct.
F	8×3 = 24 24 − 4 = 20 12 ÷ 2 = 6 12×3 = 36 − 6 30 30 leaves	Student multiplies 8 caterpillars by 3 to get 24 caterpillars, then subtracts 4 to get 20 caterpillars. Student likely recognized that 4 caterpillars is half of 8 caterpillars, because next the student finds half of 12 leaves (6 leaves). The student then multiplies 12 leaves by 3, and subtracts a "half group of leaves" from 36 to get 30 leaves

students' mathematical thinking (Jacobs, Lamb, & Philipp, 2010). Part 2 was similar to part 1, except with a different task and with different strategies (table 1). In this paper, we discuss responses to the first two prompts: (a) Describe in detail what these students did in response to the task, and (b) What did you learn about these students' mathematical understandings? Hence,

we collected data on both what the teachers attended to and interpreted in the students' strategies, and consequently the narrative frames teachers created with respect to the strategies.

In the third part of the survey, teachers revisited all six of the strategies they had previously considered, and responded to four prompts. In this paper, we discuss the analysis of the first two prompts: (a) Is there a student you would like to talk to further? If yes, which student would you like to talk to further, and why? (b) Would you be interested in discussing a particular solution with other teachers? If yes, which solution would you discuss with other teachers, and why? Hence, we collected data on whether teachers wanted to interact with a specific student or strategy, and consequently the personal frames teachers created with respect to the strategies.

Written Artifacts of Student Thinking

The mathematical tasks and written artifacts can be seen in Table 1. To differentiate among the strategies, we focus on 6 characteristics: (a) strategy type, (b) integer/non-integer ratios, (c) exhibits non-standard calculation strategies, (d) conceptually correct/incorrect, (e) correct/incorrect calculations, and (f) work includes all steps/work or is missing steps. For strategy type, we identified what type of strategy the student used according to the literature on proportional reasoning (Carney et al., 2015; Lobato & Ellis, 2010). Strategies A and D employ a unit rate strategy, in which the student finds how many pellets (or leaves) one mouse (or caterpillar) eats, and then multiplies this number by the new number of mice (or caterpillars). Strategy B employs a cross-multiplication strategy, which provides little evidence of the degree to which the student has a conceptual understanding of proportions (Cramer, Post, & Currier, 1993). Strategies C, E, and F employ different scaling strategies, where the student scales up the original ratio (strategy C), scales up and adds equal amounts (strategy E), or scales up and adds proportional amounts (strategy F).

For integer ratios, we identify whether the student used a scalar or unit rate multiplicative relationship (Carney et al., 2015), and whether that relationship was an integer ratio or non-integer ratio (Tourniaire & Pulos, 1985). For strategies A and C, the scalar and functional multiplicative relationships are integer ratios, and for strategies D and F they are non-integer ratios. We did not include strategies B or E, because strategy B does not use either the scalar or unit rate multiplicative relationship when solving, and strategy E includes additive reasoning. For the other four categories, we simply looked for whether there was evidence the strategy exhibited that characteristic or not. For example, strategy D exhibits non-standard calculations, while the others do not. Table 2 summarizes the characteristics we identified for each strategy.

Table 2: Characteristics of Strategies

Student	Strategy Type	Integer Ratios?	Correct Calc's?	Standard Calc's?	Correct Concept?	All Steps?
A	Unit Rate	Yes	No	Yes	Yes	Yes
B	Cross-Multiplication		Yes	Yes		Yes
C	Scale Up	Yes	Yes	Yes	Yes	Yes
D	Unit Rate	No	No	No	Yes	Yes
E	Scale Up w/ Adding Equal Parts		Yes	Yes	No	Yes
F	Scale Up w/ Adding Prop. Parts	No	Yes	Yes	Yes	No

Analysis

Analysis was conducted by the first author. In parts 1 and 2, teachers described the students' strategies and understandings. Hence, all teachers created narrative frames in some form as they noticed the student's work and described what the student did. Consequently, the first author

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focused solely on whether the teacher's description got the gist of the strategy, or the key underlying reasoning of the strategy. Examples include descriptions similar to our own, or descriptions of the strategy we identified (e.g. "Student A found the unit rate and solved for 24 mice"). Non-examples include identifying a strategy other than the one we identified, claiming the student was confused when evidence existed that the student was not, or admitting confusion about what the student did (e.g. "I have no idea what this student did"). More than 20% of the data was double coded by another researcher, and interrater reliability was 87%.

In part 3, teachers had a choice of talking or not talking to/about a student. However, it turned out that 100% of the teachers chose to talk to/about at least one student. The first author coded for *which* student a teacher selected, and *what* they wanted to talk about. Responses fell into four categories: (a) learn more about the strategy, (b) help the student, (c) share the strategy with the class or with other teachers, or (d) discuss other mathematical topics. More than 20% of the data was double coded by another researcher, and interrater reliability was 91%.

Finally, the first author also looked for instances when teachers spontaneously expressed excitement toward a strategy (e.g. "Love it," "This student is a genius," "Student F is my favorite"), which we took as evidence that the teacher created a personal connection with the strategy (i.e. a personal frame). Even though we did not actively seek to collect data on teachers' emotions, 24% of the teachers spontaneously expressed excitement for at least one strategy at some point in the survey. More than 20% of the data was double coded by another researcher, and interrater reliability was 93%.

Results

For narrative frames, we counted instances when the teacher's response captured the gist of the strategy. Percentages of teachers that that captured the gist of the strategy can be seen in Table 3. Almost every teacher captured the gist of strategy B, which was the cross-multiplication strategy. Comparing the correct unit rate strategies (i.e. A & D) with the correct scaling strategies (i.e. C & F), we see that the scaling strategies were more challenging for teachers to capture the gist of the strategy. Additionally, within these four conceptually correct strategies, it appears the non-integer ratio strategies (i.e. D & F) were more challenging than the integer ratio strategies (A & C), respectively, but only marginally more challenging than their counterparts. Strategy E, which included additive reasoning, was more challenging than the unit rate and cross multiplication strategies, but less challenging than the other two scalar strategies. For personal frames, we counted instances when a teacher wanted to talk to/about a particular student, and then categorized responses based on what they wanted to talk about. We also looked for evidence that a teacher enjoyed a particular strategy. Percentages can be seen in Table 3. When considering the conceptually correct scalar and unit rate strategies (i.e. A, C, D,

Table 3: Percentages of teachers capturing gist, wanting to talk to/about a student, and expressing emotion

N = 72	A	B	C	D	E	F
% of teachers that captured gist of the strategy	89%	97%	63%	83%	69%	57%
% of teachers who want to talk to/about a student	8%	15%	25%	33%	60%	75%
% of teachers expressing excitement	0%	3%	3%	11%	4%	18%

& F), scalar strategies elicited more teachers wanting to talk to/about the student than the unit rate strategies, and the strategies with non-integer ratios were more intriguing to the teachers

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than the integer-ratio strategies. Strategy F (which was the most challenging, included non-integer ratios, used a scaling up strategy, and appeared to not show all steps) elicited the largest percentage of teachers wanting to talk to/about the student, as well the largest percentage of teachers expressing excitement. Strategy E, which had the conceptual error, elicited the second highest percentage of teachers wanting to talk to/about the student. Strategy D, which had the non-standard algorithms, elicited the second highest percentage of teachers expressing excitement. Looking at what excited the teachers for strategy D, ¾ of the teachers who expressed excitement specifically mentioned the non-standard algorithms.

The percentages of which students teachers wanted to talk to/about and why can be seen in table 4[1]. Notice that almost half of the teachers wanted to learn more about strategy F, which supports the notion that strategy F was the most challenging and intriguing to teachers. Strategy F also elicited the highest percentage of teachers who wanted to share the strategy, either with their class or with other teachers.

Looking at strategy E, which had the conceptual error, we see that this strategy elicited the highest percentage of teachers (38%) wanting to help the student. In contrast, less than 10% of the teachers wanted to help students A and D (respectively), who exhibited calculational errors in their work. (The teachers who wanted to help students C and F assumed they were confused.)

When considering the conceptually correct scalar and unit rate strategies (i.e. A, C, D, & F), we see two more pieces of evidence that scalar strategies and strategies with non-integer ratios are more interesting than their counter-parts. First, more teachers wanted to learn about scalar strategies than unit-rate strategies, and more teachers wanted to learn more about the strategies with non-integer ratios than the strategies with integer ratios. Second, more teachers wanted to share the scalar strategies with others than the unit rate strategies, and more teachers wanted to share the strategies with non-integer ratios than the strategies with integer ratios. When sharing strategies, teachers often either expressed excitement for the strategy, or valued the different ways of thinking exhibited in the work.

Finally, teachers wanted to discuss a variety of other things with their peers. This included learning about some new mathematics, anticipating other solutions, discussing the importance of multiple solutions, discussing pedagogical strategies for teaching proportional reasoning, and assessment strategies. Notice that the cross multiplication strategy had a large number of teachers wanting to discuss other mathematical ideas. All of these teachers wanted to discuss whether the cross-multiplication strategy exhibited a high level of understanding, or not. This

Table 4: Percentages of teachers wanting to talk to/about a student and why

N = 72	A	B	C	D	E	F
Want to learn about strategy	3%	4%	14%	18%	22%	46%
Want to help student	4%	1%	4%	6%	38%	8%
Want to share strategy		4%	7%	4%	6%	18%
Want to discuss other topics	1%	6%	1%	7%	1%	8%
Total wanting to talk to/about student	8%	15%	25%	33%	60%	75%

means that not only did these teachers recognize that this student might have used a memorized procedure, but they were also interested in discussing such implications with other teachers.

[1] In table 4, the columns do not sum to the totals because some teachers created multiple different personal connections for the same student. Additionally, the totals do not sum to 100% because some teachers chose to talk to/about multiple students.

Discussion

To summarize, we found that some strategies elicited teachers' desire for discussions more often than others, albeit for different reasons and in different ways. In this section we discuss the strategies and the interpretive frames teachers created, and for what purposes PD leaders might select particular strategies.

Strategy F was clearly the most exciting, interesting, and challenging for teachers. We conjecture that this strategy was difficult and interesting for four reasons. First, the scalar strategy type appears to be less familiar to teachers than the unit rate or cross multiplication strategy. Second, the non-integer ratios afforded certain complexities that would not have been available in a task structure with integer ratios. In particular, this student has to both iterate and partition the ratio, and then subtract a partitioned ratio from the larger ratio (Lobato & Ellis, 2010). In strategy C, the student need only multiply the 18 pellets by a whole number; there is no need to partition the original ratio. Third, the missing step was a challenging step for many teachers to notice, but also an important part of the strategy. Finally, the computations are all quite simple at first glance. We wonder if the simple computations created a slight misdirection in teachers' expectations as they considered the student's strategy. Perhaps the excitement teachers expressed arose from prevailing in understanding a challenging strategy.

Student E was also often chosen by teachers, but for different reasons than for student F. Many of those who wanted to talk to/about student E wanted to help student E fix the error, or develop a stronger understanding. Contrasting this with the other two strategies that exhibited errors, strategy A and strategy D, we see that the conceptual error in strategy E was more interesting to teachers than the calculational errors in strategies A and D. Hence, it appears conceptual errors intrigue teachers more than calculational errors.

Strategies D and F elicited the most excitement from teachers. For strategy D, ¾ of the teachers who exhibited excitement specifically cited the non-standard division and/or multiplication algorithms as exciting. Strategy F appeared to be exciting in general. Considering the two strategies, we highlight four similarities that may have supported such excitement. In particular, strategy F and the non-standard algorithms of strategy D were correct, based on underlying concepts, complex, and unfamiliar to teachers. We conjecture that other strategies with these qualities may also elicit excitement from teachers.

Another idea that emerged from the data was that 6% of the teachers wanted to discuss the underlying mathematics of strategy B, and whether it counted as a deep conceptual understanding or not. It appears that these teachers recognized that the student might have been following a memorized procedure, and wanted to discuss with other teachers what this might mean with respect to learning and teaching. We believe that conversations like the one these teachers wanted to have could be productive for teachers. Perhaps other more traditional strategies, when surrounded by non-standard or conceptually-based strategies, could spark conversations among teachers about the underlying mathematics.

In their study of video artifacts of student thinking, Sherin, et al. (2009) rated videos based on three particular characteristics, and looked for which types of videos supported conversations among teachers. They concluded that there wasn't a particular characteristic that was more important than others, but rather certain combinations of characteristics supported better discussions. Our results seem to support this idea, that no single characteristic makes a solution interesting, but rather a combination of characteristics.

Earlier we mentioned that we recognized there were important differences between the two teacher groups from which we collected our data. In our analysis we noticed that there were

indeed differences among the teacher groups. For example, the four teachers who wanted to discuss whether the cross multiplication strategy exhibited a deep conceptual understanding were all practicing teachers. In contrast, the two teachers who wanted to show off the cross multiplication strategy (because they valued the strategy) to other teachers were prospective teachers. However, due to the scope of this paper we did not aim to parse out these differences, and instead focused on investigating what interpretive frames our teachers invoked and how they were invoked across the different strategies. In future work we will disaggregate our data and investigate differences in interpretive frames among the teacher groups.

We end by emphasizing that different artifacts serve different purposes. Each strategy seemed to have its own unique set of challenges, and teachers wanted to respond to different ideas based on the different strategies. In our work we only looked at 6 different strategies and the qualities associated with these strategies, and aggregated data from a diverse group of secondary teachers. We wonder what other kinds of strategies researchers might consider, what other combinations of qualities might pique teachers' interest, and how teachers' interests differ across different populations of teachers.

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